Relativistic pondermotive Hamiltonian for electrons in an intense laser field

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A Hamiltonian theory of the electron drift motion in an intense laser pulse is presented. The action-variational Lie perturbation method is utilized to derive the relativistic pondermotive Hamiltonian in a rigorous and systematic way. The results include: the electron drift motion in a linearly polarized pulse is slightly anisotropic because of the finite pulse duration effects, and its drift in a circularly polarized pulse contains a vortex component. [S1063-651X(99)11805-1]

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The concept of a pondermotive force is one of the basic tools of plasma physics because of its numerous applications. It describes the slower drift motion of the particle oscillating center [1] after averaging over the fast quiver motion of the particle. It has been studied by several authors before [2–4], where the electromagnetic fields are treated as a small perturbation to the particle's free motion. The corresponding averaged equation of motion for the electron is written

$$\frac{d\overline{\mathbf{p}}}{dt}=-\nabla\Phi_p=-\frac{e^2}{2m\omega^2}\nabla|\overline{\mathbf{E}}|^2, \tag{1}$$

where $\mathbf{p}$ is the electron momentum, $-e$ the electron charge, m the electron mass, ω the laser frequency, and $\mathbf{E}$ the electric field, where the overbar denotes an average over the laser period.

In the electromagnetic field of an ultraintense laser [5], the electron motion becomes highly relativistic and the electromagnetic fields play a dominant role in electron dynamics at the lowest order. Obviously the concept of pondermotive force needs to be generalized and its validity domain examined. Previous studies [6–9] of this problem have provided a basic physical picture of the so-called relativistic pondermotive potential: the averaged electron motion is isotropic and electrons are expelled from the high-intensity regions. One undesirable character of their methods is that they are not systematic, therefore very difficult to use to derive higher order results, which are qualitatively different from the lowest order ones in the present case.

In this paper, a different way of treating electron drift motion in an intense laser pulse is adopted. It is well known that the motion of charged particles in a plane wave is exactly integrable [10]. Meanwhile a tightly focused short laser pulse can be accurately described as a traveling wave with slowly changing amplitude and phase [9,11]. These observations suggest that an adiabatic perturbation theory is applicable to particle motion in a laser pulse at least in some portion of particle phase space. As will be shown later, the Lagrangian describing the motion of nonresonant particles can be separated into a zero order, oscillation-free part and a smaller part containing fast oscillation terms. After an averaging over the fast oscillation, the drift motion of the electrons in terms of the ''oscillation-center'' variables can be described by the pondermotive Hamiltonian. This averaging process can be most easily done using the action-variational Lie perturbation method [12,13]. This method preserves the Hamiltonian structure of the original system which is a prerequisite for all proper extension to the treatment of many-particle systems such as plasmas through the methods of statistical mechanics and kinetic theory. It provides a systematic way of deriving the pondermotive Hamiltonian, in principal, to all orders. Unlike the nonrelativistic regime, in which the pondermotive Hamiltonian is itself a second order quantity, its relativistic counterpart is a zero order quantity and its higher order corrections can be important for applications. For example, we will show that the first order pondermotive Hamiltonian for a short linearly polarized laser pulse is dependent on its polarization direction, which is in contrast to the zero order pondermotive Hamiltonian.

The interaction of electrons with an intense laser pulse can be described by the following Hamiltonian in terms of the canonical variables $(\mathbf{q},\mathbf{p})$:

$$h=\sqrt{m^2c^4+c^2[\mathbf{p}+e/c\mathbf{A}(\mathbf{q})]^2}, \tag{2}$$

where c is the velocity of light and $\mathbf{A}(\mathbf{q})$ is the vector potential of the laser field. For a finite-length tightly focused laser pulse, its vector potential has the following form [9]:

$$\begin{aligned}\mathbf{A}=&\frac{mc^2}{e}\mathbf{a}_\perp+\epsilon\frac{mc^2}{e}\mathbf{a}_z+O(\epsilon^2)=\frac{mc^2}{e}\mathbf{b}_\perp(\epsilon\mathbf{r}_\perp,\epsilon^2z,\sigma\xi)e^{ick\xi}\\&+\epsilon\frac{mc^2}{e}\mathbf{b}_z(\epsilon\mathbf{r}_\perp,\epsilon^2z,\sigma\xi)e^{ick\xi}+O(\epsilon^2)+\text{c.c.},\end{aligned} \tag{3}$$

where $\xi=t-z/c$ measures the distance behind the leading edge of the pulse and k is the laser wave vector in vacuum. Two smallness parameters, $\epsilon=1/kw_0$ and $\sigma=/kc\Delta\tau$, where w_0 is the beam waist at focus and $\Delta\tau$ is the pulse duration, have been introduced to explicitly show the ordering of different terms. Physical results will be obtained by setting them to 1. The ϵ and σ in front of the coordinates denote the order of the terms produced by differentiation with corresponding coordinates. Because of the finite pulse duration effect, both $\mathbf{b}_\perp$ and b_z are infinite power series in increasing powers of σ. Note the smallness of σ is not essential in our treatment, since as long as $\mathbf{A}$ is a function that depends only on ξ, the electron equation of motion is exactly integrable. Therefore $\mathbf{A}$ can have any functional dependence on ξ, and a perturbative analysis of one form or another is still applicable. In our calculation, the smallness of σ is used to express the results in simple form. The explicit form of $O(\epsilon^2)$ terms in the vector potential will only be needed when we calculate the pondermotive potential to the order of $O(\epsilon^3)$, and is therefore not needed for present purposes.

Hamiltonian equations of motion can be obtained from the variation of the action

$$\delta S=\delta\int(\mathbf{p}\cdot d\mathbf{r}-h\,dt)=0 \tag{4}$$

which can also be put in the form

$$\delta S=\delta\int[\mathbf{p}_\perp\cdot d\mathbf{r}_\perp+(p_z-h/c)dz-h(dt-dz/c)]. \tag{5}$$

Equation (5) can be interpreted as an equation similar to Eq. (4) in which $\xi=t-z/c$ plays the role of time, h is the Hamiltonian and p_z-h/c is the momentum conjugate to z. Writing $j_z=p_z+\epsilon a_z-h/c$ and eliminating p_z gives

$$h=-\frac{j_z}{2}-\frac{1+(\mathbf{p}_\perp+\mathbf{a}_\perp)^2}{2j_z}, \tag{6}$$

where we have taken the units $m=c=e=1$. The purpose of this transformation is that the new momentum $(\mathbf{p}_\perp,j_z)$ are constants of motion for the ''unperturbed'' Hamiltonian, namely, Eq. (1) with $\mathbf{A}$ being the vector potential of a plane wave, which is convenient for a perturbation analysis of the particle motion (when it is applicable) in a realistic laser pulse similar to Eq. (3).

The covariance of the variational formulation under an arbitrary phase space coordinate transformation is manifested more clearly by considering the fundamental one-form of Poincaré-Cartan

$$\gamma=\gamma_\mu dz^\mu=\mathbf{p}_\perp\cdot\mathbf{r}_\perp+j_z dz-\epsilon a_z dz-h\,d\xi, \tag{7}$$

where $z^0=t,\ \gamma_0=-h$. Under the coordinate transformation $\overline{z}=\overline{z}(z,\xi)$ we then have

$$\gamma=\gamma_\mu dz^\mu=\overline{\gamma}_\mu d\overline{z}^\mu, \tag{8}$$

where $\overline{\gamma}_\mu=\gamma_\sigma\partial z^\sigma/\partial\overline{z}^\mu$. The variation of γ yields the Euler-Lagrange equation

$$\omega_{\mu\nu}\frac{dz^\nu}{d\xi}=0, \tag{9}$$

where $\omega_{\mu\nu}=\partial\gamma_\nu/\partial z^\mu-\partial\gamma_\mu/\partial z^\nu$ is the Lagrange tensor. It should be noted that the Euler-Lagrange equation is invariant under the gauge transformation $\gamma\rightarrow\gamma+dS$, for any scalar function S on the extended phase space.

Our purpose is to eliminate the fast oscillation of Eq. (7) through an averaging procedure. It is conceptually simpler and probably more physical to proceed through a number of steps systematically. First, we look for a phase-space coordinate transformation and a gauge function that will eliminate the fast quiver motion in the leading order of γ (denoted by γ_0), provided it is possible. We then include the smaller part of γ and find another transformation that removes the fast quiver motion, order by order, from it.

When the radiation field is a plane wave, the following generating function $W(\mathbf{r},\mathbf{P})$ will bring the Hamiltonian to a form free of oscillating terms:

$$W=\mathbf{r}\cdot\mathbf{P}+\frac{1}{J_z}\int^\xi\mathbf{P}_\perp\cdot\mathbf{a}_\perp d\xi'+\frac{1}{2J_z}\int^\xi\{a_\perp^2\}d\xi', \tag{10}$$

where $\{a_\perp^2\}=b_\perp^2e^{2ik\xi}+\text{c.c.}$ denotes the oscillating part of $a_\perp^2$. This suggests that the following phase-space coordinate transformations are helpful in separating the drift motion from the quiver motion:

$$\mathbf{r}_\perp=\mathbf{R}_\perp-\frac{1}{J_z}\int^\xi\mathbf{a}_\perp(\mathbf{R},\xi')d\xi',$$

$$z=Z+\frac{1}{J_z^2}\int^\xi\mathbf{P}_\perp\cdot\mathbf{a}_\perp(\mathbf{R},\xi')d\xi'+\frac{1}{2J_z^2}\int^\xi\{a_\perp^2(\mathbf{R},\xi')\}d\xi',$$

$$\mathbf{p}_\perp=\mathbf{P}_\perp,$$

$$j_z=J_z, \tag{11}$$

where $(\mathbf{R}_\perp,Z,\mathbf{P}_\perp,J_z)$ are the oscillation-center variables at the lowest order. For a plane wave propagating in vacuum, they are the exact canonical momentum and coordinates of the particle oscillation center which has a uniform drift motion with velocity $d\mathbf{R}_\perp/d\xi=-\mathbf{P}_\perp/J_z,\ dZ/d\xi=-1/2+(1+\mathbf{P}_\perp^2+2|\mathbf{a}_\perp|^2)/(2J_z^2)$. In the laser field given by Eq. (3), they are not canonical conjugate variables anymore, and the particle dynamics is much more complicated. For future use, we define the oscillation radius vector $\boldsymbol{\rho}=\boldsymbol{\rho}_\perp+\rho_z\mathbf{e_z}=-(1/J_z)\int^\xi\mathbf{a}_\perp(\mathbf{R},\xi')d\xi'+[(1/J_z^2)\int^\xi\mathbf{P}_\perp\cdot\mathbf{a}_\perp(\mathbf{R},\xi')d\xi'+(1/2J_z^2)\int^\xi\{a_\perp^2(\mathbf{R},\xi')\}d\xi']\mathbf{e_z}+\text{c.c.}$

Before we determine the fundamental one-form (Lagrangian) in the noncanonical coordinates $(\mathbf{R}_\perp,Z,\mathbf{P}_\perp,J_z)$, it is appropriate to discuss which part of particle phase space the perturbation methods are applicable to. As far as the particle motion is concerned, a realistic pulse of Eq. (3) can be considered ''slightly'' different from the plane wave only when the particle oscillation amplitude is much smaller than the characteristic scale lengths of the laser envelope (which are infinity for a plane wave), i.e., during one oscillation period, the particle experiences an almost uniform field. This is equivalent to the requirement that the particle oscillation amplitude changes very little during a single fast oscillation. Only then does the implicit assumption that the integrable solution of the new system exists actually hold, which is necessary for a perturbative treatment. This will effectively exclude the resonant particles, namely those with ''very small'' j_z. More precisely, the maximum excursion of the particle from its oscillation center in the perpendicular direction must satisfy the following: $|\rho_\perp|\simeq a_\perp/(j_zk)\ll w_0$, and therefore j_z must satisfy $j_z\gg\epsilon a_\perp$. Note that the particle excursion in the z direction is less important in determining the validity domain of the perturbation analysis, because the characteristic length of the laser envelope in the z direction is one order larger than the perpendicular characteristic length. It is worth pointing out that under the above condition, Eq. (11) can be inverted order by order.

Using Eqs. (7) and (8), and adding a scalar function $S = (-1/J_z)\int^{\xi}\mathbf{P}_\perp\cdot\mathbf{a}_\perp(\mathbf{R},\xi')d\xi' - (1/J_z)\int^{\xi}\{a_\perp^2(\mathbf{R},\xi')\}d\xi'$, we have

$$\gamma = \gamma_0 + \epsilon\gamma_1 + O(\epsilon^2),$$

$$\gamma_0 = \mathbf{P}_\perp\cdot d\mathbf{R}_\perp + J_z dZ + [J_z/2 + (1+\mathbf{P}_\perp^2 + 2|\mathbf{a}_\perp|^2)/(2J_z)]d\xi,$$

$$\begin{aligned}\gamma_1 = {}& a_z/J_z\nabla_{\mathbf{P}_\perp}G\cdot\mathbf{dP}_\perp + 2a_z/J_z\partial G/\partial J_z\cdot dJ_z + \nabla_\perp G\cdot\mathbf{dR}_\perp\\ &- a_z dZ + [(1/J_z)\nabla_\perp(\mathbf{P}_\perp\cdot\mathbf{a}_\perp)\cdot\boldsymbol{\rho}_\perp + (1/2J_z)\nabla_\perp(\mathbf{a}_\perp)^2\cdot\boldsymbol{\rho}_\perp\\ &+ a_z/J_z\partial G/\partial\xi]d\xi,\end{aligned} \tag{12}$$

and $G = -(1/J_z)\int^{\xi}(\mathbf{P}_\perp\cdot\mathbf{a}_\perp + 1/2\{a_\perp^2\})d\xi$. In the above calculation, we have expanded the vector potential appearing in the Hamiltonian as $\mathbf{a}(\mathbf{R}+\boldsymbol{\rho}) = \mathbf{a}(\mathbf{R}) + \epsilon\partial\mathbf{a}/\partial\mathbf{R}_\perp\cdot\boldsymbol{\rho}_\perp + O(\epsilon^2)$, which is only possible for nonresonant particles. We see once more the difference between resonant and nonresonant particle dynamics. The motion of a resonant particle cannot be separated into a quiver part and a drift part that can be described by a Lagrangian free of oscillation terms. For a nonresonant particle, though, this separation can clearly be done; the drift motion is well described by the pondermotive Hamiltonian and the oscillation motion can be obtained from a coordinate transformation.

Because of the adiabatic ordering, we must take the variation of both γ_0 and γ_1 to determine the momentum equation correct to order ϵ. Therefore the negative of the time component of γ_0 cannot yet be interpreted as the relativistic pondermotive potential at the lowest order in the sense of Eq. (1). Only when there exists a coordinate transformation that will eliminate all the terms in γ_1 except for $\gamma_{1\xi}$ exists, can $-\gamma_{0\xi}$ be interpreted as the lowest order relativistic pondermotive potential for the corresponding transformed coordinates.

We can now use Lie transformation whose application had been discussed in detail in Refs. [12] and [13] to eliminate the fast "time" ξ dependence from γ_1. Lie transformation provides a systematic and elegant way to carry out the averaging procedure to higher order, though here we stop at first order, which already produces some nontrivial results. We seek a transformation from $(\mathbf{R}_\perp, Z, \mathbf{P}_\perp, J_z)$ to $(\overline{\mathbf{R}}_\perp, \overline{Z}, \overline{\mathbf{P}}_\perp, \overline{J}_z)$, where we use an overbar to indicate the fast time-averaged phase-space coordinates.

The Lie coordinate transform can be formally written as

$$Z^\mu = Tz^\mu, \tag{13}$$

where $T = \exp[-\int^{\epsilon}d\lambda L(\lambda,\epsilon)]$. L is the Lie operator with $L(\lambda,\epsilon) = \Sigma\lambda^{n-1}L_n(\epsilon), L_n(\epsilon) = L_{n0} + \epsilon L_{n1} + \epsilon^2 L_{n2}$. Based on the ordering for the laser envelope, the Lie operator acts on scalar f and one-form γ as

$$L_n f = g_n^\mu\partial f/\partial z^\mu = L_{n0}f + \epsilon L_{n1}f + \epsilon^2 L_{n2}f,$$

$$L_{n0}f = g_n^{\mathbf{p}}\frac{\partial f}{\partial\mathbf{p}} + g_n^{\xi}\frac{\partial f}{\partial\xi},$$

$$L_{n1}f = g_n^{\mathbf{r}_\perp}\frac{\partial f}{\partial\mathbf{r}_\perp},$$

$$L_{n2}f = g_n^{z}\frac{\partial f}{\partial z}, \tag{14}$$

$$L_n\gamma = g_n^{\nu}\left(\frac{\partial\gamma_\mu}{\partial z^\nu} - \frac{\partial\gamma_\nu}{\partial z^\mu}\right)dz^\mu = L_{n0}\gamma + \epsilon L_{n1}\gamma + \epsilon^2 L_{n2}\gamma,$$

$$L_{n0}\gamma = \left(g_n^{\mathbf{p}}\frac{\partial\gamma_\mu}{\partial\mathbf{p}} + g_n^{\xi}\frac{\partial\gamma_\mu}{\partial\xi}\right)dz^\mu - g_n^{\nu}\frac{\partial\gamma_\nu}{\partial\mathbf{p}}d\mathbf{p} - g_n^{\nu}\frac{\partial\gamma_\nu}{\partial\xi}d\xi,$$

$$L_{n1}\gamma = g_n^{\mathbf{r}_\perp}\frac{\partial\gamma_\mu}{\partial\mathbf{r}_\perp}dz^\mu - g_n^{\nu}\frac{\partial\gamma_\nu}{\partial\mathbf{r}_\perp}d\mathbf{r}_\perp,$$

$$L_{n2}\gamma = g_n^{z}\frac{\partial\gamma_\mu}{\partial z}dz^\mu - g_n^{\nu}\frac{\partial\gamma_\nu}{\partial z}dz, \tag{15}$$

where g_n^μ is the generator of Lie transform.

On the other hand, under the transformation T, the one form γ becomes

$$\Gamma = T\gamma + dS, \tag{16}$$

where S represents a gauge transformation in phase space. Upon expanding Γ, γ, S, and T in powers of ϵ, we have

$$\Gamma_0 = \gamma_0, \tag{17}$$

$$\Gamma_1 = \gamma_1 - L_{10}\gamma_0 + \frac{\partial S_1}{\partial\mathbf{p}}d\mathbf{p} + \frac{\partial S_1}{\partial\xi}d\xi, \tag{18}$$

$$\begin{aligned}\Gamma_2 = {}& \gamma_2 - L_{10}\gamma_1 - (L_{11} - 1/2L_{10}^2 + 1/2L_{20})\gamma_0 + \frac{\partial S_1}{\partial\mathbf{r}_\perp}\cdot d\mathbf{r}_\perp\\ &+ \frac{\partial S_2}{\partial\mathbf{p}}d\mathbf{p} + \frac{\partial S_2}{\partial\xi}d\xi,\end{aligned} \tag{19}$$

where for simplicity we take $S_0 = 0$. The perturbation calculation consists of finding the transformation order by order by specifying S_n and g_n^μ in a way that simplifies Γ_μ. A second expansion in powers of σ will be used in each order to express the results in simple terms.

Now we are ready to consider the γ given by Eq. (12). We do not transform "time" ξ, so we take $g_n^\xi = 0$. From Eqs. (15) and (18), we have

$$\begin{aligned}\Gamma_1 = {}& (\nabla_\perp G - g_1^{\mathbf{P}_\perp})\mathbf{dR}_\perp + (-a_z - g_1^{J_z})dZ\\ &+ (\nabla_{\mathbf{P}_\perp}S_1 + g_1^{\mathbf{R}_\perp} + a_z/J_z\nabla_{\mathbf{P}_\perp}G)\mathbf{dP}_\perp\\ &+ (\partial S_1/\partial J_z + g_1^{Z} + 2a_z/J_z\partial G/\partial J_z)dJ_z\\ &+ \{\partial S_1/\partial\xi - g_1^{\mathbf{P}_\perp}\cdot\mathbf{P}_\perp/J_z - g_1^{J_z}\\ &\times[1/2 - (1+\mathbf{P}_\perp{}^2 + 2|\mathbf{a}_\perp|^2)/(2J_z)] + \gamma_{1\xi}\}d\xi.\end{aligned} \tag{20}$$

In the above equation, we can choose S_1 and g_1^μ such that all $\Gamma_{1\mu}$ vanish except $\Gamma_{1\xi}$. Also, by requiring that there be no fast oscillating term in $\Gamma_{1\xi}$, we obtain

$$\Gamma_{1\xi}=\langle\gamma_{1\xi}\rangle=\left\langle\frac{1}{J_z}\nabla_\perp(\mathbf{P}_\perp\cdot\mathbf{a}_\perp)\cdot\boldsymbol{\rho}_\perp-\frac{1}{J_z^2}a_z(\mathbf{P}_\perp\cdot\mathbf{a}_\perp)\right\rangle. \tag{21}$$

Using the fact $\nabla\cdot\mathbf{a}=0$ so that $a_z=\int^\xi(\nabla_\perp\cdot\mathbf{a}_\perp)d\xi+O(\epsilon^2)=-J_z\nabla_\perp\cdot\boldsymbol{\rho}_\perp+O(\epsilon^2)$, we can express the result as

$$\Gamma_{1\xi}=1/J_z\nabla_\perp\cdot\langle[(\mathbf{P}_\perp\cdot\mathbf{a}_\perp)\boldsymbol{\rho}_\perp]\rangle$$
$$=-\frac{1}{J_z^2}\nabla_\perp\left\langle\left[(\mathbf{P}_\perp\cdot\mathbf{a}_\perp)\int^\xi\mathbf{a}_\perp(\mathbf{R},\xi')d\xi'\right]\right\rangle. \tag{22}$$

For the same reason as before, in order to derive the momentum equation correct to order ϵ^2, we need to find S_2 and g_2^μ to eliminate all the components of γ_2 except $\gamma_{2\xi}$. This can be done by following the same procedure as above [13]. For our purposes, it is probably more important to know that such a coordinate transformation exists than to explicitly write it.

Upon adding Eqs. (17) and (22) we set $\epsilon=1$ and $\sigma=1$, and obtain the fundamental one form, which describes the particle motion in fast-time averaged coordinates $(\overline{\mathbf{R}}_\perp,\overline{Z},\overline{\mathbf{P}}_\perp,\overline{J}_z)$ in the electromagnetic fields of a tightly focused short pulse:

$$\Gamma=\overline{\mathbf{P}}_\perp\cdot d\overline{\mathbf{R}}_\perp+\overline{J}_z d\overline{Z}-Hd\xi, \tag{23}$$

where H is the relativistic pondermotive Hamiltonian

$$H=H_0+H_1,$$

$$H_0=-\overline{J}_z/2-(1+\overline{\mathbf{P}}_\perp{}^2+2|\mathbf{b}_\perp|^2)/(2\overline{J}_z),$$

$$H_1=-\frac{1}{\overline{J}_z^2}\nabla_\perp\left\langle\left[(\overline{\mathbf{P}}_\perp\cdot\mathbf{a}_\perp)\int^\xi\mathbf{a}_\perp(\overline{\mathbf{R}},\xi')d\xi'\right]\right\rangle$$
$$=\frac{1}{\overline{J}_z^2}\nabla_\perp[1/(ik)(\overline{\mathbf{P}}_\perp\cdot\mathbf{b}_\perp)\mathbf{b}_\perp^*-1/(ik)(\overline{\mathbf{P}}_\perp\cdot\mathbf{b}_\perp^*)\mathbf{b}_\perp$$
$$-1/(k^2)(\overline{\mathbf{P}}_\perp\cdot\mathbf{b}_\perp)\partial\mathbf{b}_\perp^*/\partial\xi-1/(k^2)(\overline{\mathbf{P}}_\perp\cdot\mathbf{b}_\perp^*)\partial\mathbf{b}_\perp/\partial\xi$$
$$+O(\sigma^2)]. \tag{24}$$

The smallness of σ has been used in expressing H_1 in simple terms.

The Euler-Lagrange equations resulting from the variation of Eq. (23) are

$$d\overline{\mathbf{R}}/d\xi=\partial H_0/\partial\overline{\mathbf{P}}+\partial H_1/\partial\overline{\mathbf{P}}+O(\epsilon^2),$$
$$d\overline{\mathbf{P}}_\perp/d\xi=-\partial H_0/\partial\overline{\mathbf{R}}_\perp-\partial H_1/\partial\overline{\mathbf{R}}_\perp+O(\epsilon^3),$$
$$d\overline{J}_z/d\xi=-\partial H_0/\partial Z-\partial H_1/\partial Z+O(\epsilon^4). \tag{25}$$

Here H_0 is the lowest order relativistic pondermotive Hamiltonian. It gives the main features of relativistic electron drift motion [6–9]: for a linearly polarized pulse, it is independent of the laser polarization direction, and the electrons are expelled from the high-density regions.

H_1 represents the first order correction to the relativistic pondermotive Hamiltonian. For a short linearly polarized laser pulse, $\mathbf{b}_\perp=b_\perp\mathbf{e}_x$, the leading term of H_1 disappears and

$$H_1=-\frac{1}{k^2\overline{J}_z^2}\frac{\partial}{\partial\overline{x}}(\overline{P}_x\partial|b_\perp|^2\partial\xi).$$

We see that H_1 has a weak dependence (of the order of $\epsilon\sigma$) on the pulse polarization direction. The electron drift velocities in the perpendicular plane are given by

$$v_{Dx}=d\overline{X}/d\xi=-\overline{P}_x/\overline{J}_z-\frac{1}{k^2\overline{J}_z^2}\frac{\partial}{\partial\overline{x}}(\partial|b_\perp|^2\partial\xi),$$

$v_{Dy}=d\overline{Y}/d\xi=-\overline{P}_y/\overline{J}_z$, which manifestly show that because of the finite pulse duration effects, electron drift motion is slightly anisotropic. To elaborate on the discussion of isotropy, let us examine the electron perpendicular drift motion in cylindrical coordinates (R,Θ,Z). The angular drift velocity is given by

$$d\Theta/d\xi=-\frac{P_\Theta}{J_z\overline{R}^2}+\frac{\sin(2\Theta)}{k\overline{R}J_z^2}\frac{\partial}{\partial\overline{R}}(\partial|b_\perp|^2/\partial\xi).$$

Of interest is the second term: It causes the electrons to move towards the x axis, which is the laser polarization direction- ,during the arising edge of the pulse, and vice versa during the falling edge but with a smaller amount since then the electron is in the relatively lower intensity region. So the net effect on the electron drift motion is that while electron is moving towards lower intensity region, it also slightly rotate towards the laser polarization axis. However, the amount of this anisotropy is too small to explain the observed strong anisotropy of electron scattering from laser pulse in the recent experiment [14].

For a circularly polarized pulse, $\mathbf{b}_\perp=b_\perp(\mathbf{e}_x+i\mathbf{e}_y)$, $H_1=-(2/k\overline{J}_z^2)[(\overline{\mathbf{P}}_\perp\times\nabla_\perp|b_\perp|^2)\cdot\mathbf{e}_z]+O(\epsilon\sigma)$, $\mathbf{v}_D=-\overline{\mathbf{P}}_\perp/\overline{J}_z-(2/k\overline{J}_z^2)\nabla_\perp\times|b_\perp|^2\mathbf{e}_z$. We see that the electron drift contains a vortex component in this case.

This formalism can be easily extended to study the electron drift motion in vacuum beat wave configuration [15]. Here two beams with slightly different frequency propagate at small angles of θ and $-\theta$ with respect to the z axis. The essential requirement for our treatment to be valid is that the variation of the vector potential in all but one of the coordinates is slow. This is obviously satisfied by vacuum beat wave configuration. In fact, a pondermotive Hamiltonian of the same form as Eq. (24) can be derived, with the properly chosen $\mathbf{b}_\perp$.

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